A REPUTATION GAME (CAMPAIGNING FOR RESPECT)

Bob   John

■ ————————► "Challenge"   ■ ——————— ■   (-1,-1)   or (-1,2)

                                                                if John   if John
                                                                 "soft"   "tough"

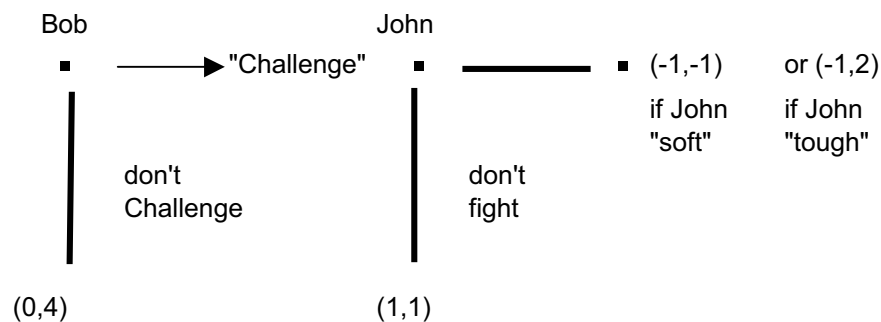don't   don't
Challenge   fight

(0,4)   (1,1)

Payoffs =(Bob, John)

Let Π = Bob's estimate of the probability that John is "tough"

If the game is played only once,

- "tough" John always fights; "soft" John never fights.

- So , Bob challenges if and only if $\Pi < 1/2$

But this is not true if the game is repeated – played twice. In the repeated game , even a "soft" John may fight to gain a reputation for toughness. [So, game theory can illuminate the "campaign for respect" point made by Anderson]

Repeated Reputation Game

(*) Suppose the Reputation Game is to be played twice between the same two players, with the same payoffs as before.

(*) By observing what John does in the first play of the game, Bob may learn whether John is "tough" or "soft.

(*) But, John knows this, and so may try to create a "reputation for toughness" . But, Bob knows that John knows this , and will take this into account…

We know the following ["tough" John always fights when challenged.]

(1) "soft" John not fighting for sure in the first period cannot be an equilibrium.

. <u>why?</u> Because, then any time Bob sees John fight in the first period , he will know John must be "tough" and so Bob will not challenge in second period. But, then it would pay for "soft" John to fight in the first period in order to avoid being challenged in the second period.

(2) If $\Pi<1/2$ (so Bob challenges in one shot game) then "soft" John fighting for sure in the first period also cannot be an equilibrium.

.<u>why?</u> Because if John always fights, whether "soft" or "tough" then Bob can learn nothing from the first period play, and so Bob will challenge for sure. There would be no point in "soft" John fighting in the first period (since he can't avoid being challenged by doing so.)

(3) If $\Pi<1/2$, then not challenging for sure in the second period in response to John's fighting in the first period cannot be an equilibrium.

.<u>why?</u> Beause then "soft" John would fight for sure in the first period. (since $-1+4>1+1$)

Therefore the equilibrium must be as follows.

(a) "tough John always fights when challenged.

(b) "soft" John never fights in the second period.

(c) Bob always challenges in the second period if John doesn't fight in the first period

(d) "soft" John "mixes" randomly between fighting and not fighting in the first period. Bob "mixes" randomly between challenging and not challenging in the second period, in response

"mixing" is like fastball or curve, down the line or cross court.

Full Solution for Two Period Game

(*) Let ρ= probability "soft" John fights in the first period.

γ= probability Bob challenges in the second period after being fought in the first period.

(*) Then "soft" John "mixes" in the first period only if

payoff if John doesn't fight = payoff if John does fight, i.e

$1+1=-1+4(1-γ)+1*γ$

so γ=1/3

(*) Also, Bob "mixes" in the second period only if

payoff if Bob doesn't challenge = expected payoff if Bob challenges in the second period. i.e

0=1*Probability["fighting" John is soft]-1*Probability["fighting" John is tough]

So, Probability["fighting" John is soft]=1/2=(1-Π)ρ/(Π+(1-Π)ρ)

(using Bayes' Rule)

Hence ρ=Π/(1-Π)

Also, Π<1/4 is necessary for Bob to challenge in the first period. (⇒ρ<1/3)

Mixing in between 2 different actions in a Game

Baseball Game

Pitcher

		Throw a Fast Ball	Throw a Curve Ball
Batter	Look for a Fast Ball	(1,-1)	(-1,1)
	Look for a Curve Ball	(-1,1)	(1,-1)

Payoff = (Batter, Pitcher)

(*) No equilibrium exist if either player taking some action for sure. [since, when opponent responds, player would always want to change]

(*) So, both players must "mix". Hence, both must be indifferent between their two actions.

(*) Hence, in this game, equilibrium involves

Batters looks for Fast Ball with probability $=1/2$

Pitcher throws a Fast Ball with probability $=1/2$